Partitioned Successive-Cancellation Flip Decoding of Polar Codes

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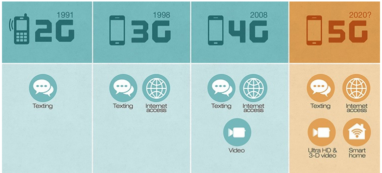
Motivation



- **Polar Codes** provably achieve channel capacity
  - Adopted in **5G eMBB** control channel
  - Being considered for **5G URLLC & mMTC** channels
- **5G** standardization targets
  - Improved error-correction performance, T/P
  - Low power/energy consumption

[1] E. Arıkan. ”Channel polarization: A method for constructing capacity-achieving codes for symmetric binary-input memoryless channels,” IEEE Transactions on Information Theory, vol. 55, no. 7, pp. 30513073, July 2009.

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- Successive Cancellation Flip **(SC-Flip)** decoding

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- Successive Cancellation Flip **(SC-Flip)** decoding
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  - ✓ Implementation complexity close to that of SC
  - ✓ Average latency converges to that of SC

SC-Flip Decoding: Idea

- Classifying erroneous decisions into:
  - Channel-induced errors
  - Propagated errors due to a previous error

[2] O. Afisiadis, A. Balatsoukas-Stimming and A. Burg, ”A low-complexity improved successive cancellation decoder for polar codes,” 2014 48th Asilomar Conference on Signals, Systems and Computers, Pacific Grove, CA, 2014, pp. 2116-2120.

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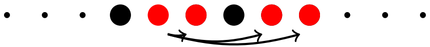

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  - SC decoding is supported by a CRC
  - Multiple SC iterations are necessary

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Example: $N = 16$, $K + C = 12$, $T_{max} = 4$

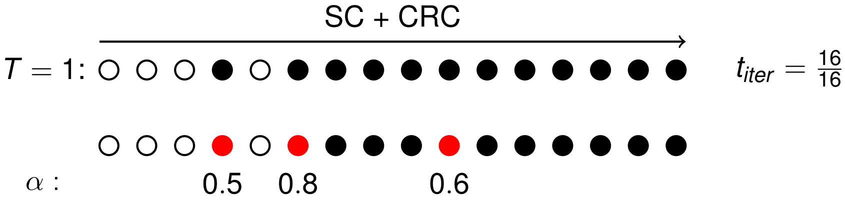
- First iteration: pass/fail?

$\xrightarrow{\text{SC + CRC}}$

$T = 1$: ○ ○ ○ ● ○ ● ● ● ● ● ● ● ● ● ● ● $t_{iter} = \frac{16}{16}$

Example: $N = 16$, $K + C = 12$, $T_{max} = 4$

- Fail: Flip $T_{max} - 1$ least reliable indices, one at a time



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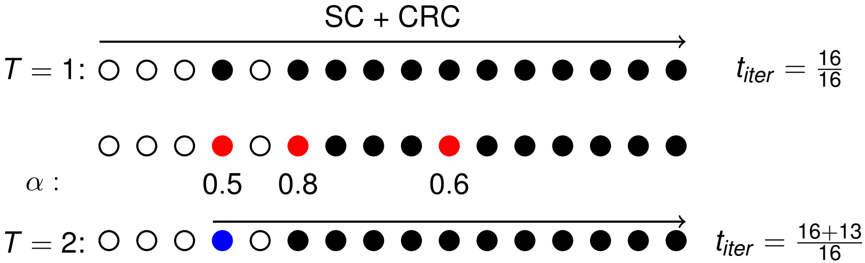
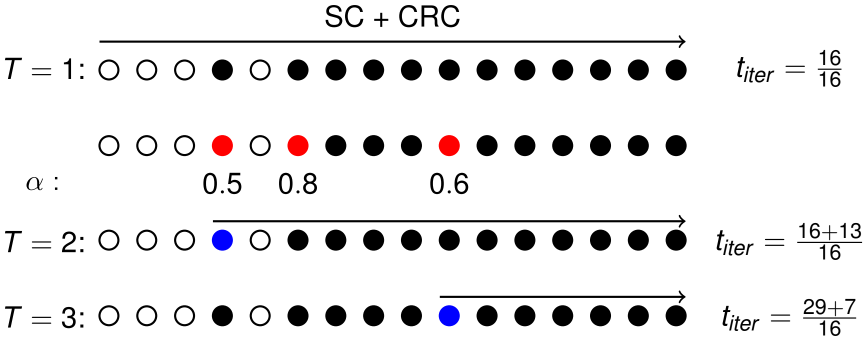
- $T = 2$: Flip least reliable index



Example: $N = 16$, $K + C = 12$, $T_{max} = 4$

- $T = 3$: Flip second least reliable index



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$t_{iter} = \frac{29+7}{16}$

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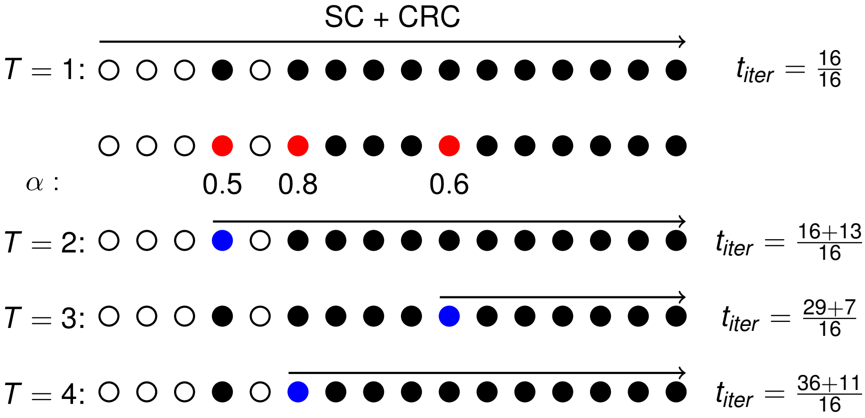
- $T = 4$: Flip third least reliable index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SC-Flip Decoding: Insights

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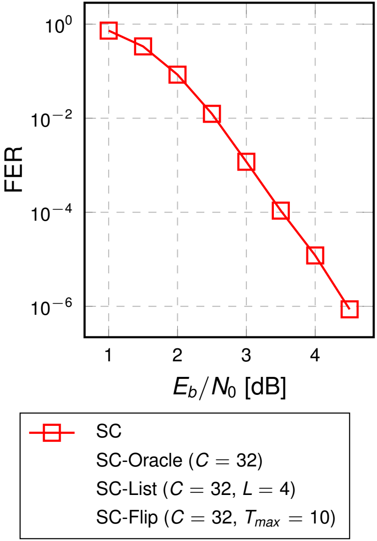

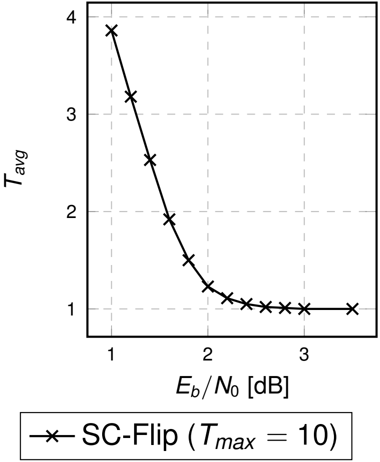

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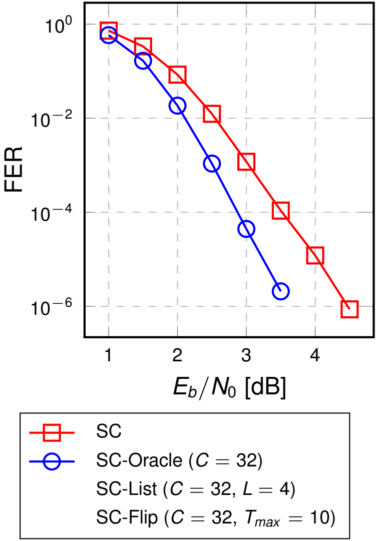

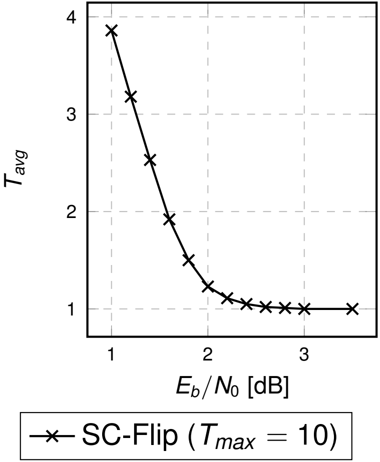

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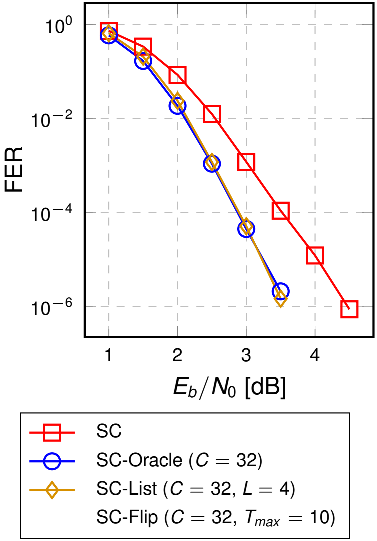

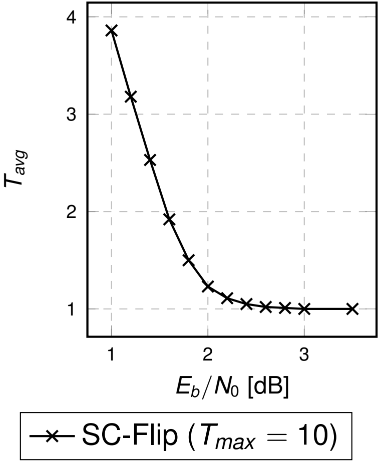

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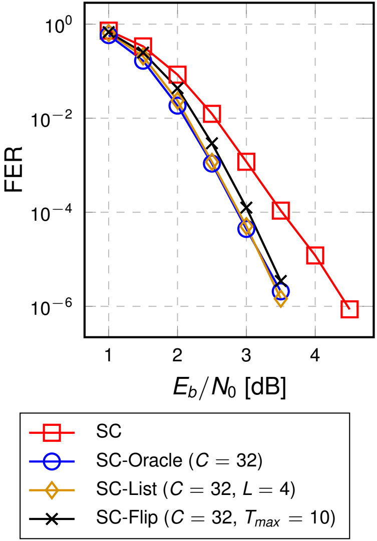

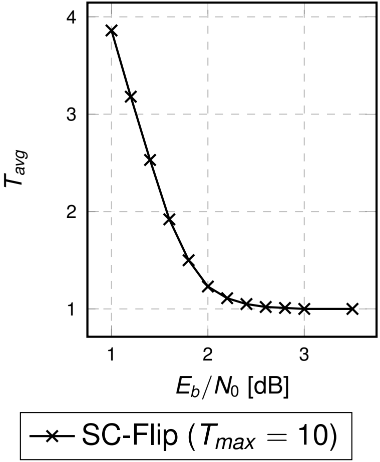

Partitioned SC-Flip Decoding Algorithm (PSCF)

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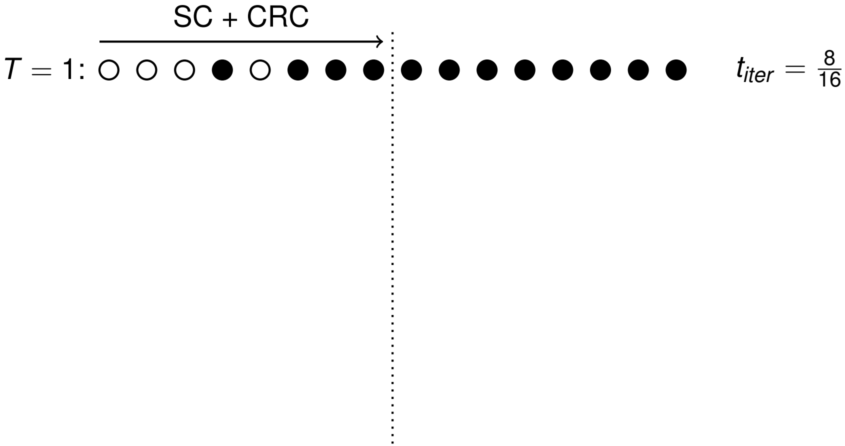
  - ✓ **Reduced iterations, enables early termination**
- ✓ Possibility of correcting **multiple channel-induced errors**
  - ! ...if each error resides in a different partition

PSCF Decoding Algorithm

- Codeword is divided into sub-blocks


SC + CRC
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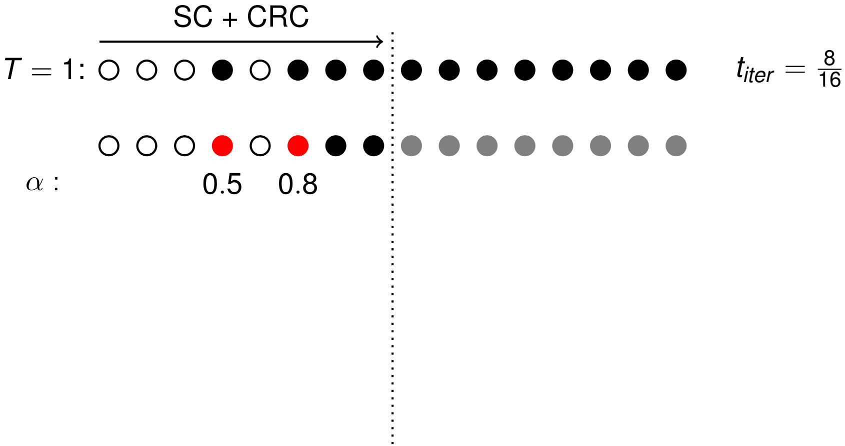

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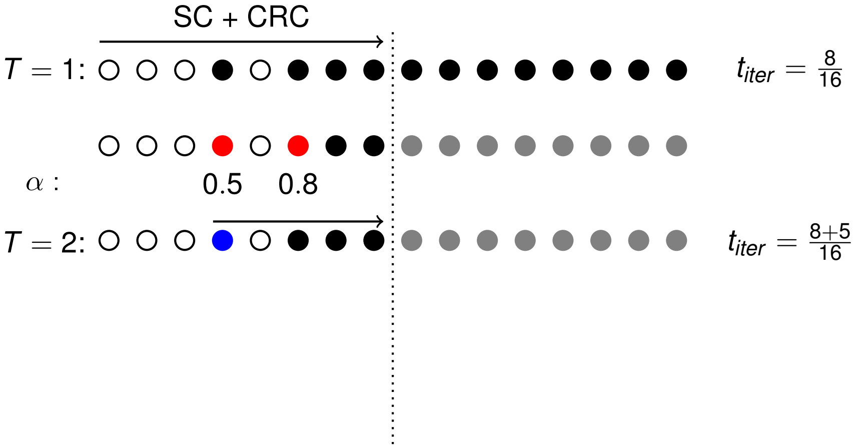

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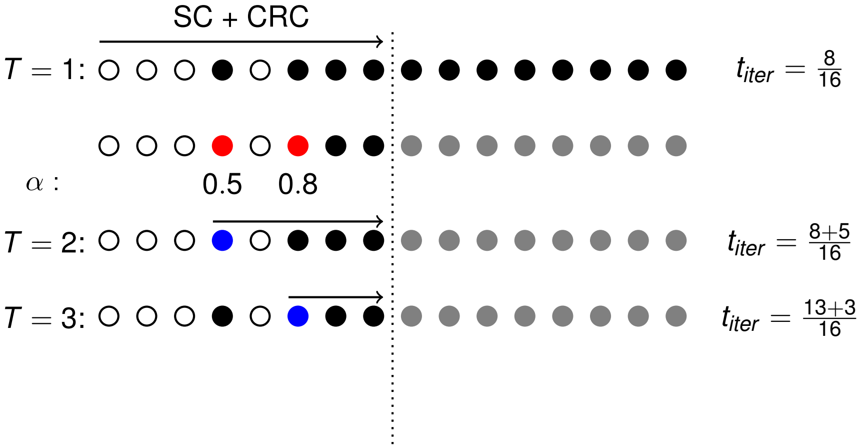

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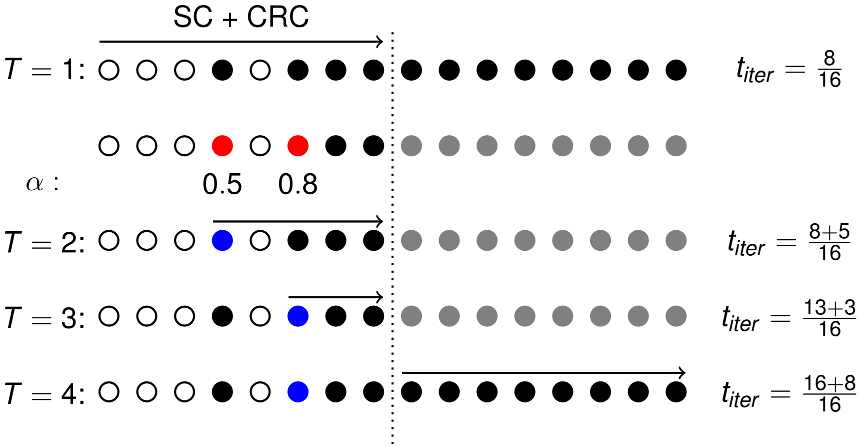
PSCF Decoding Algorithm

- Shorter iterations result reduced T_{avg}



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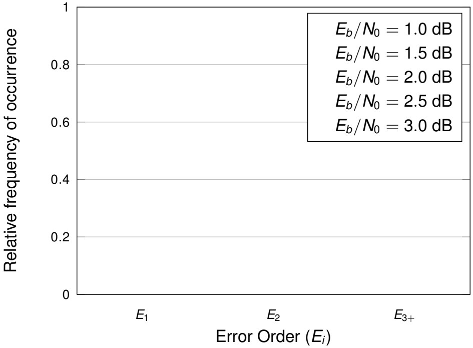
- Objective: to cover equal amount of error probability in each partition



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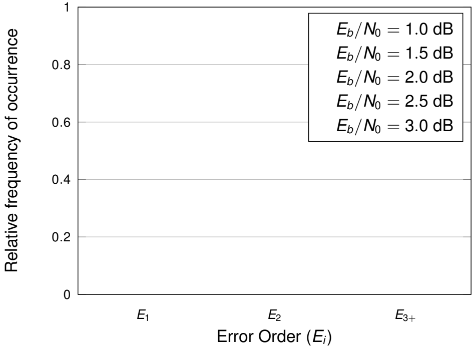

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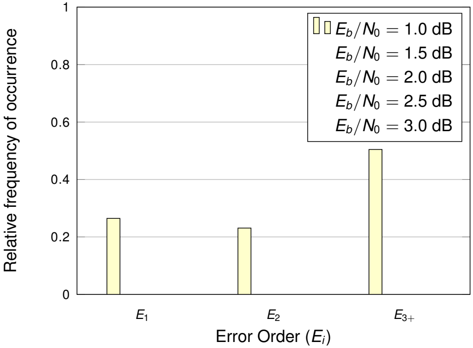

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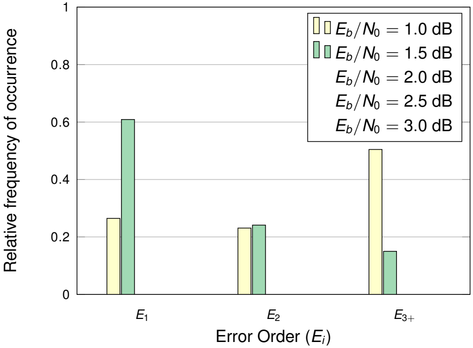

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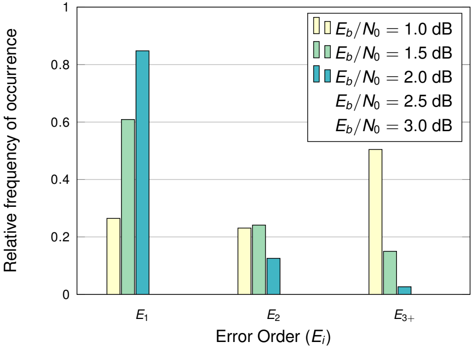

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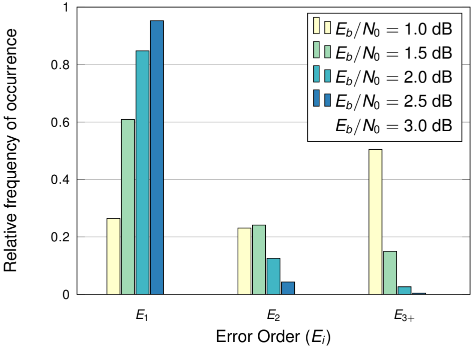

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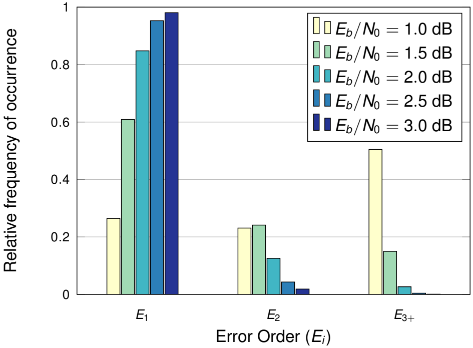

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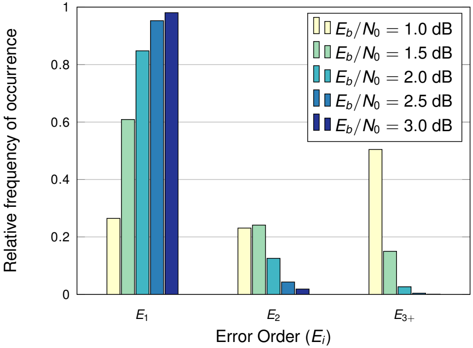

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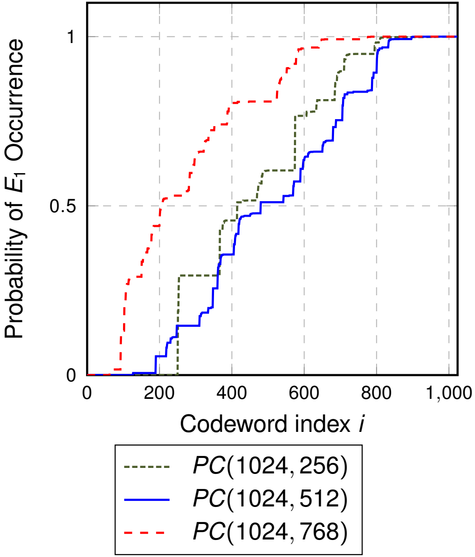
- E_1 dominates the error occurrence at medium/high SNR → can be used to approximate partitioning

Codeword Partitioning

- Figure: CDF based on E_1 ($E_b/N_0 = 2.5$ dB)



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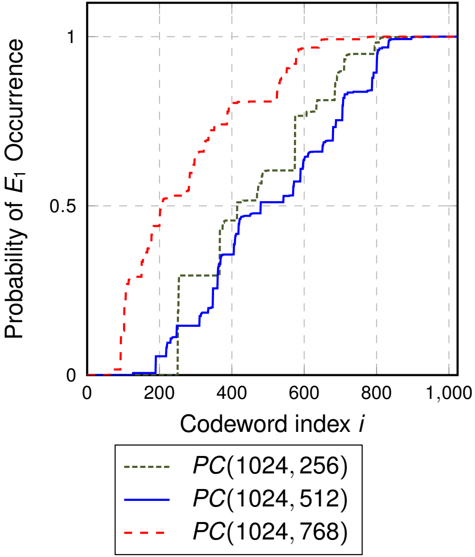

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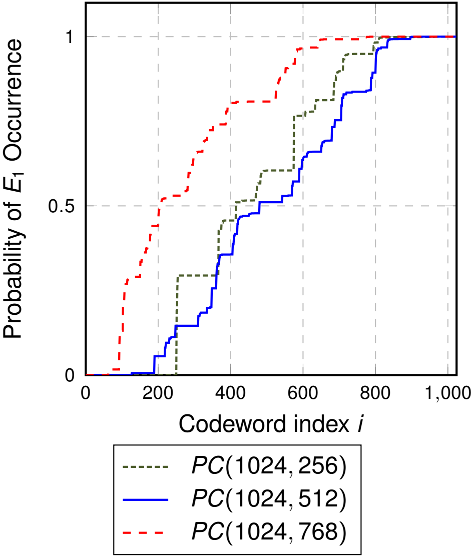

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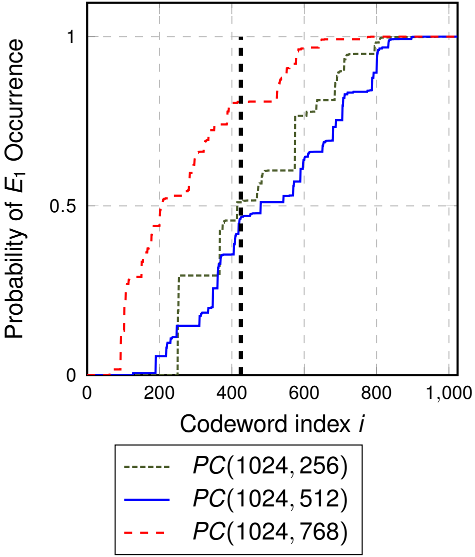

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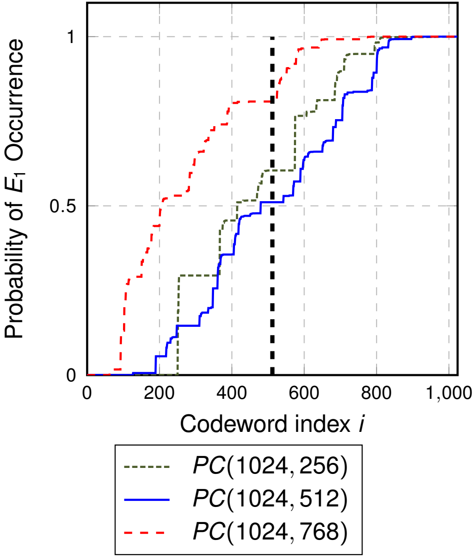

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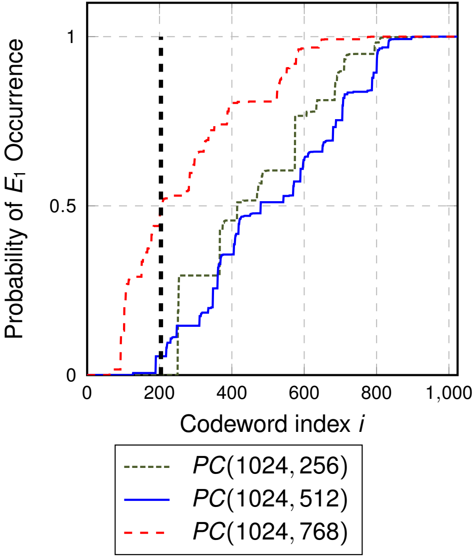

Simulation Results - Performance

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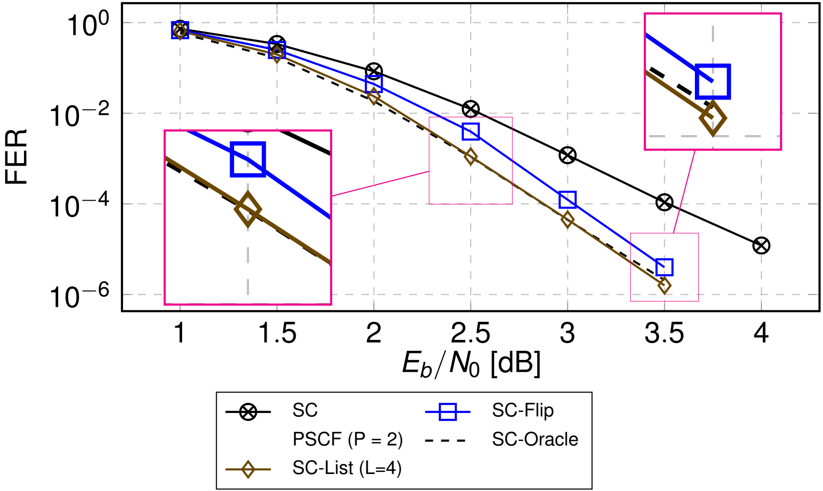

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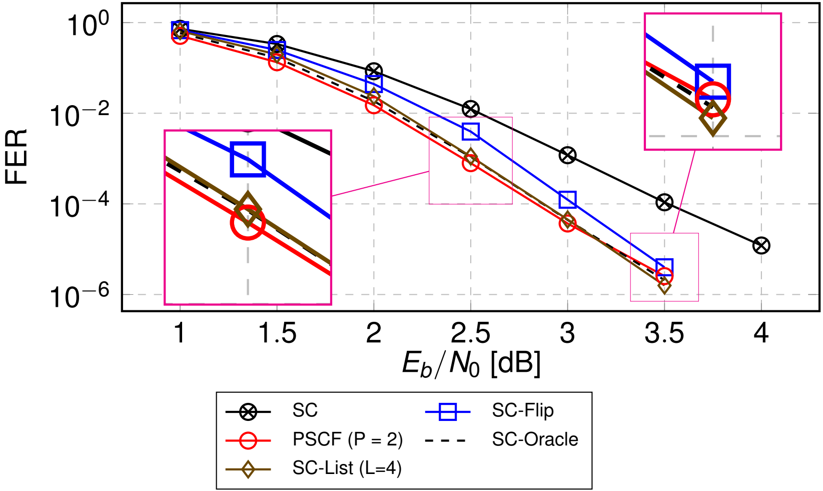
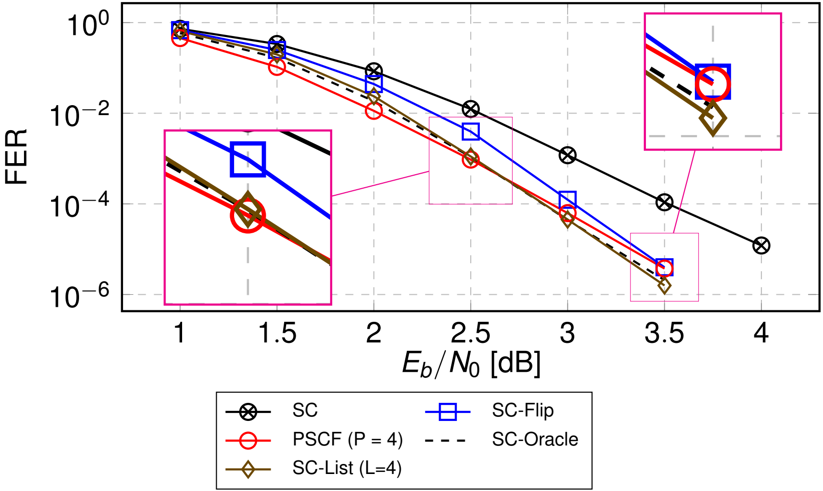


Simulation Results - Performance (cont'd)

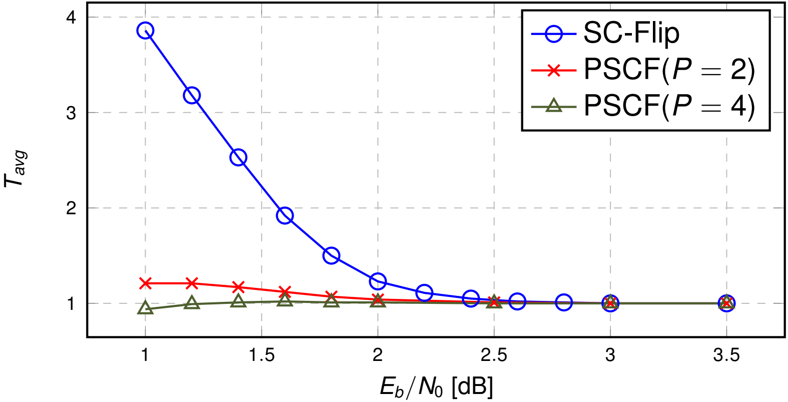
- $PC(1024, 512)$, $C = 32$, $T_{max} = 10$, $P = 4$



Simulation Results - Average Iterations

- $PC(1024, 512)$, $C = 32$

- At matching FER with SC-Flip ($T_{max} = 10$)



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  - ✓ Reduction by up to $3.2\times$ with $P = 2$ and $4.1\times$ with $P = 4$
  - ✓ Leads to **increased average throughput** and **reduced energy consumption**

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  - ✓ Improved error-correction performance by up to 0.26 dB with $P = 2$ compared to SC-Flip
- Significantly reduced T_{ave} compared to baseline SC-Flip
  - ✓ Reduction by up to $3.2\times$ with $P = 2$ and $4.1\times$ with $P = 4$
  - ✓ Leads to **increased average throughput** and **reduced energy consumption**
- Cummulative error distribution schemes help decide better partitioning

Thank you!